Design of Analog CMOS Integrated Circuits

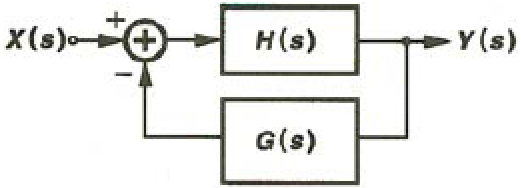
<Chapter 8>

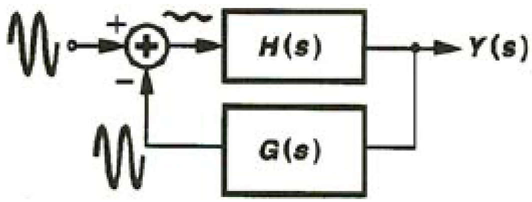
Feedback

양병도

8.1 General Considerations

❑ Negative Feedback





$$Y(s) = H(s)[X(s) - G(s)Y(s)]$$

$$\frac{Y(s)}{X(s)} = \frac{H(s)}{1 + G(s)H(s)}$$

Error signal

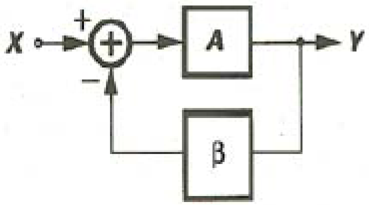
$H(s)$: *open-loop* transfer function

$Y(s)/X(s)$: *close-loop* transfer function

$H(s)$ represents an amplifier

$G(s)$: feedback factor (β), frequency-independent.

❑ feedback 특징: Gain이 공정 온도에 덜 민감해진다.

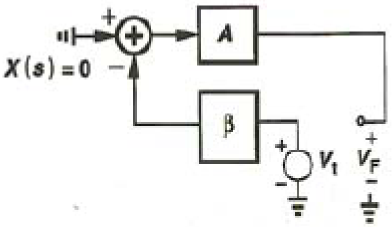


Closed-loop Gain

$$A_{CL} = \frac{Y}{X} = \frac{A}{1 + A\beta} = \frac{1}{\beta} \cdot \frac{A\beta}{1 + A\beta} \approx \frac{1}{\beta}$$

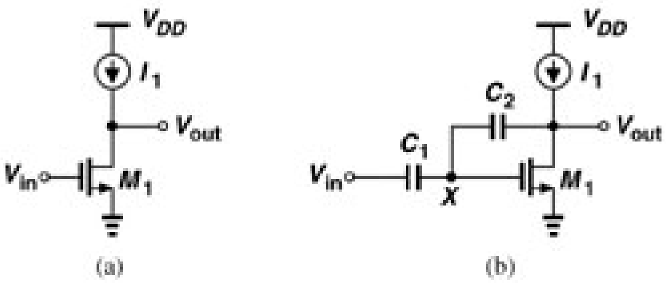
If the loop gain Aβ >> 1

Computation of loop gain:



$$\frac{V_F}{V_t} = -\beta A$$

❑ Example: CS Amplifier with feedback



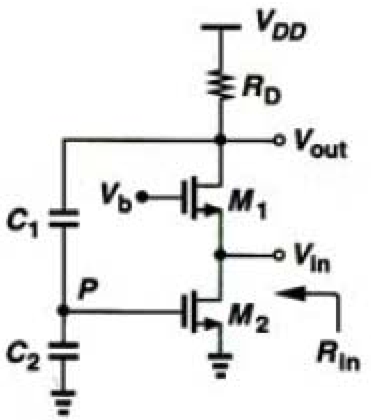
- ✓ $V_{out} / V_{in} = -g_{m1}r_{o1}$
- ✓ 온도와 공정에 의하여 g_{m1} 와 r_{o1}가 변한다.
- ✓ Gate bias voltage → not shown
- ✓ 출력에서 C_2의 영향을 무시하면, $V_{out} = - g_m V_X r_{o1}$
- ✓ $(V_{out}-V_X)C_2 s = (V_X-V_{in})C_1 s$
- ✓ 커패시터 비율은 온도와 공정에 영향을 덜 받는다.

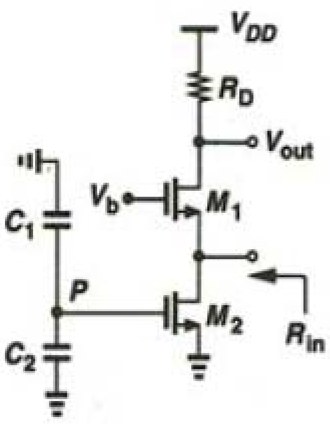
$$\frac{V_{OUT}}{V_{IN}} = -\frac{1}{\left(1+\frac{1}{g_{m1}r_{o1}}\right)\frac{C_2}{C_1}+\frac{1}{g_{m1}r_{o1}}}$$

$$= -\frac{C_1}{C_2} \quad , \quad \text{for large } g_{m1}r_{o1}$$

❑ feedback 특징: impedance가 바뀐다.

- ✓ CG circuit with feedback (λ=0)

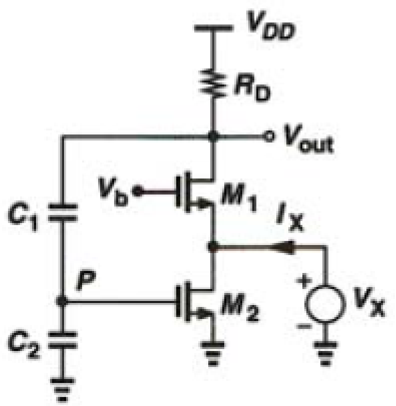




Open-loop circuit

$$R_{in,open} = \frac{1}{g_{m1} + g_{mb1}}$$

- ✓ Calculation of R_{in}

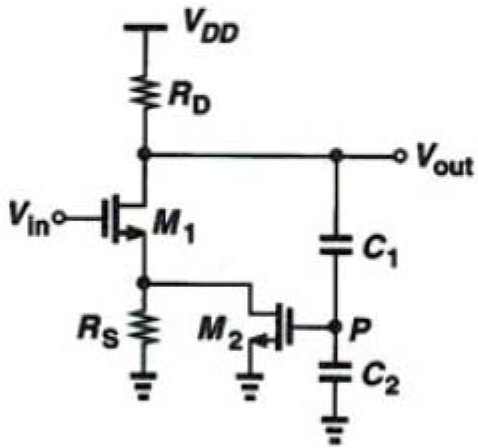


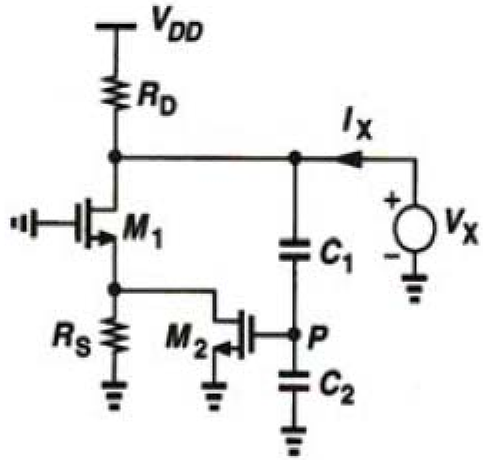
$$V_{out} = (g_{m1} + g_{mb1}) V_X R_D, \quad V_P = V_{out} \frac{C_1}{C_1 + C_2} = (g_{m1} + g_{mb1}) V_X R_D \frac{C_1}{C_1 + C_2}$$

$$I_X = (g_{m1} + g_{mb1}) V_X + g_{m2} V_P = (g_{m1} + g_{mb1}) \left(1 + g_{m2} R_D \frac{C_1}{C_1 + C_2}\right) V_X$$

$$R_{in,closed} = \frac{V_X}{I_X} = \frac{1}{g_{m1} + g_{mb1}} \frac{1}{1 + g_{m2} R_D \dfrac{C_1}{C_1 + C_2}} = R_{in,open} \frac{1}{1 + A_v \beta}$$

- ✓ CS stage with feedback
    - ➢ Calculation of Rout (λ=0)





$$V_P = \frac{C_1}{C_1 + C_2} V_{out}$$

$$I_{D2} = g_{m2} V_P = g_{m2} V_{out} \frac{C_1}{C_1 + C_2}$$

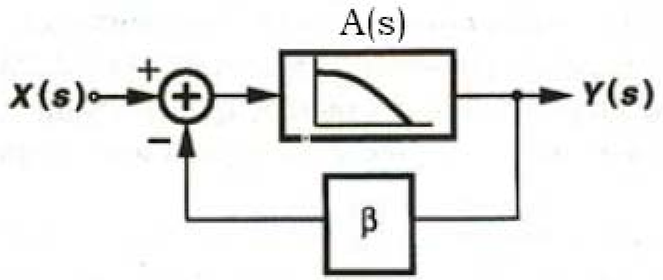
$$I_{D1} = I_{D2} \cdot \frac{R_S}{R_S + \frac{1}{g_{m1} + g_{mb1}}} = g_{m2} V_X \frac{C_1}{C_1 + C_2} \cdot \frac{R_S}{R_S + \frac{1}{g_{m1} + g_{mb1}}}$$

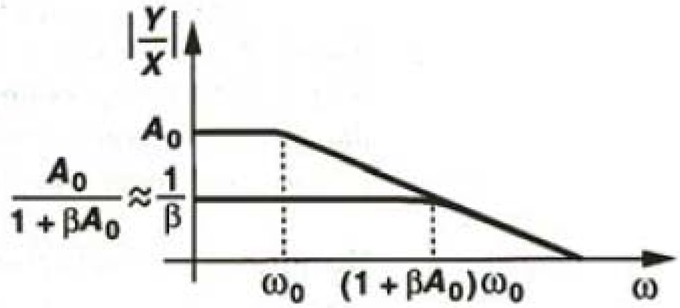
Since $I_X = V_X / R_D + I_{D1}$, we have

$$R_{out,closed} = \frac{V_X}{I_X} = \frac{R_D}{1 + \frac{g_{m2} R_S (g_{m1} + g_{mb1}) R_D}{(g_{m1} + g_{mb1}) R_S + 1} \frac{C_1}{C_1 + C_2}} = \frac{R_{out,open}}{1 + A_v \beta}$$

❑ feedback 특징: Bandwidth가 바뀐다.

✓ Bandwidth modification as a result of feedback





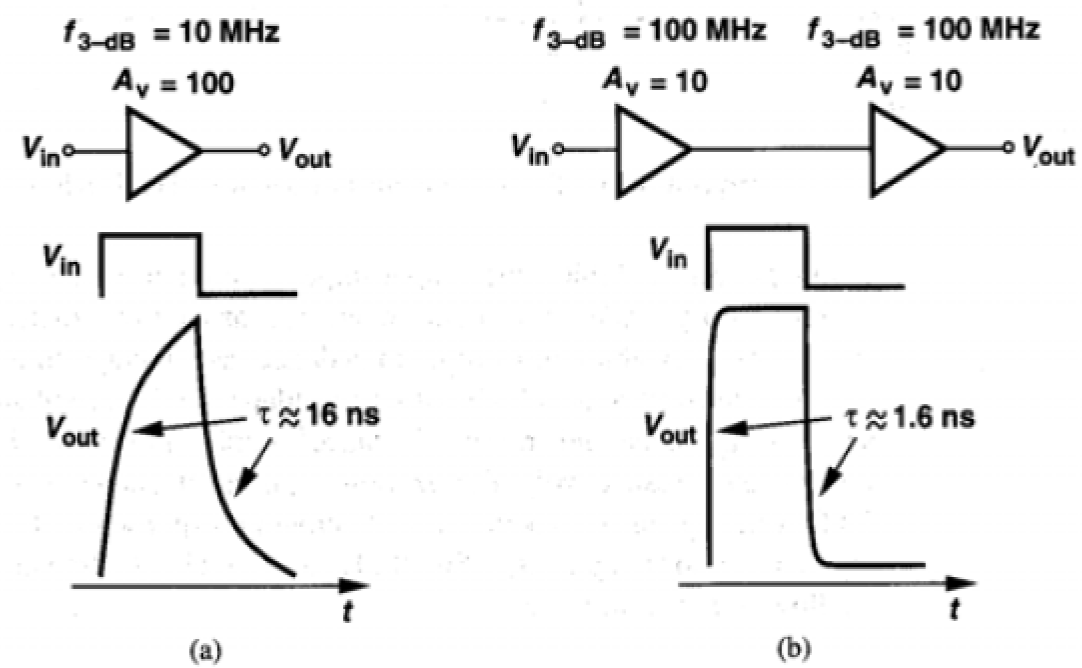
$$A(s) = \frac{A_0}{1 + \frac{s}{\omega_0}}$$

$$\frac{Y}{X}(s) = \frac{A(s)}{1 + \beta A(s)} = \frac{\frac{A_0}{1 + \beta A_0}}{1 + \frac{s}{(1 + \beta A_0)\omega_0}}$$

- Gain 감소 → $A_0 \ / (1 + \beta A_0)$
- 3dB 주파수 증가 → $(1 + \beta A_0)\omega_0$

❑ Example: Amplification of a 20-MHz square-wave

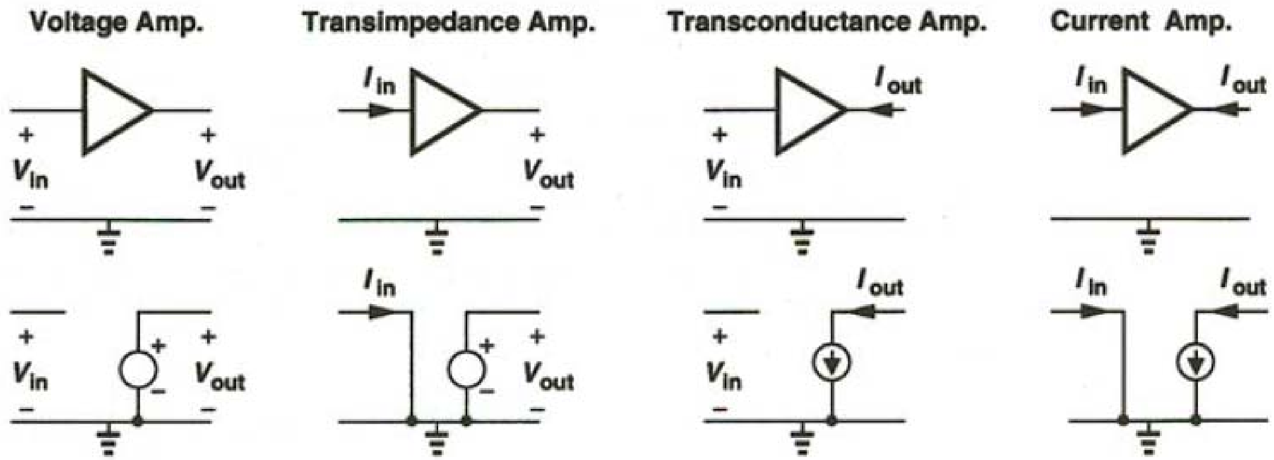
- ✓ (a) 20-MHz amplifier → Av=100, f_{3dB}=10MHz로 신호 왜곡
- ✓ (b) cascade → feedback으로 만들어진 Av=10, f_{3dB}=100MHz의 amp를 두개 사용하여 전체 gain=100, f_{3dB}=100MHz로 신호 전달



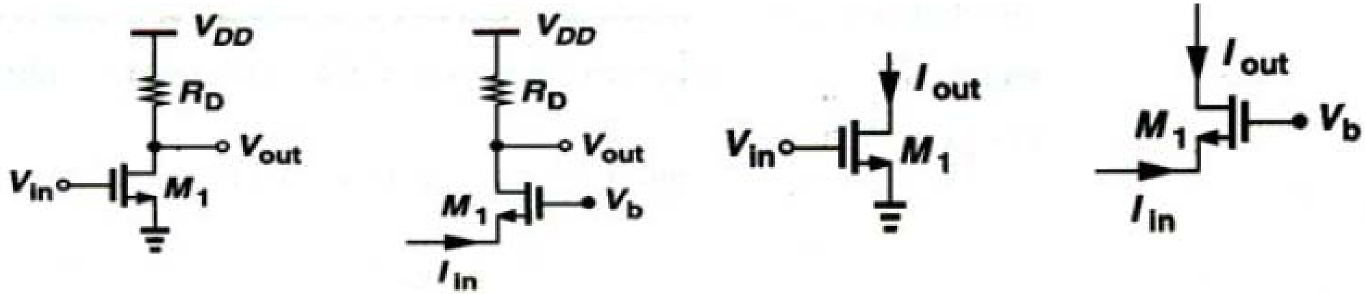
$$\tau = \frac{1}{2\pi f_{3-dB}}$$

❑ Types of amplifiers

- ✓ 다른 형태의 amp에서도 feedback 사용 가능하다.



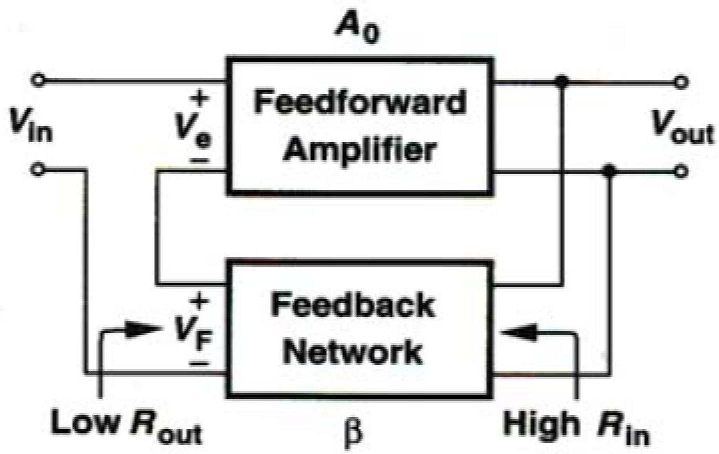
- ✓ Simple implementations of four types of amplifiers



- ✓ Voltage-voltage type이 가장 많이 쓰인다.

8.2.1 Voltage-voltage feedback

- Block diagram: series-shunt feedback



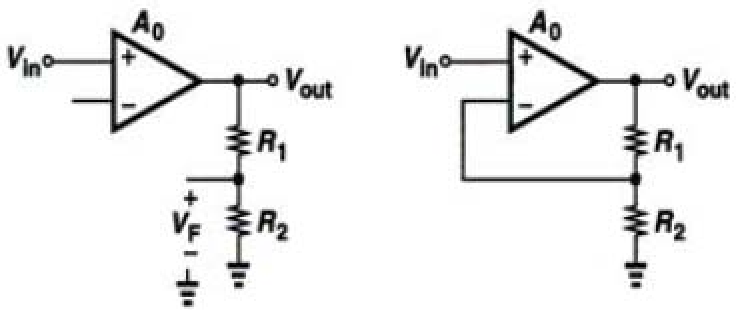
$V_F = \beta V_{out}$, $V_e = V_{in} - V_F$,

$V_{out} = A_0(V_{in} - \beta V_{out})$,

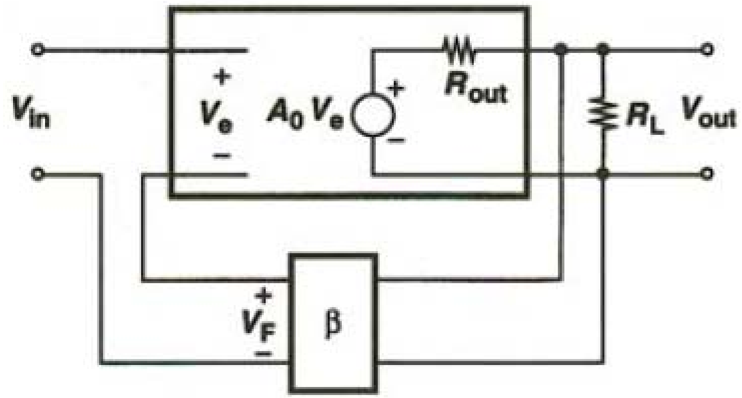
and hence $$\frac{V_{out}}{V_{in}} = \frac{A_0}{1+\beta A_0}$$

The overall gain is dropped by $1 + \beta A_0$.

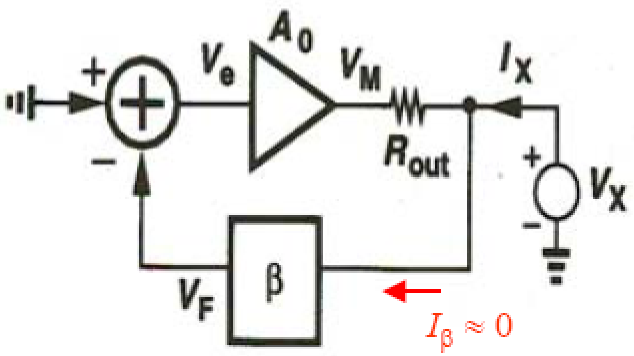
- Example



❑ Effect of voltage-voltage feedback on output resistance



❑ Calculation



$V_F = \beta V_X$, $V_e = -V_F$, $V_M = A_0 V_e = -\beta A_0 V_X$, and hence

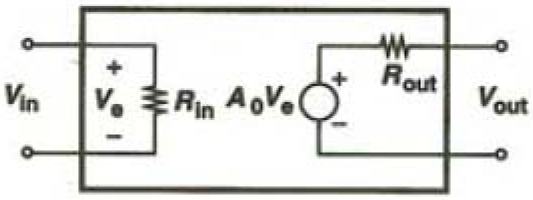
$I_X = (V_X - V_M)/R_{out} = [V_X - (-\beta A_0 V_X)]/R_{out}$. (if $I_\beta \approx 0$)

If follows that

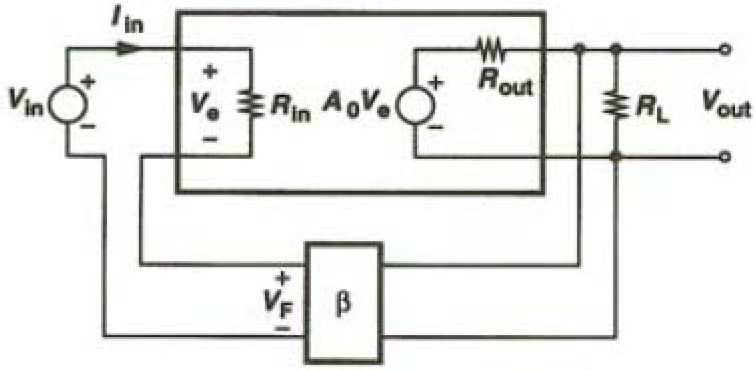
$$\frac{V_X}{I_X} = \frac{R_{out}}{1+\beta A_0}$$

The output impedance is lowered by $1 + \beta A_0$.

❑ effect of voltage-voltage feedback on input resistance

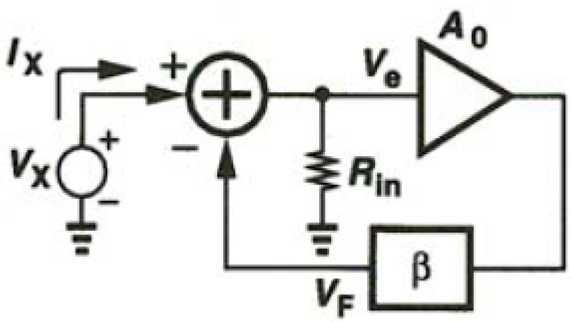


(a)



(b)

❑ Calculation



Since $V_e = I_X R_{in}$ and $V_F = \beta A_0 I_X R_{in}$ we have

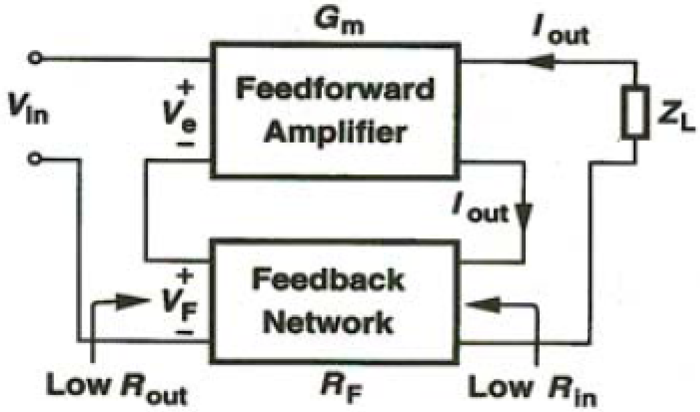
$V_e = V_X - V_F = V_X - \beta A_0 I_X R_{in}$.

Thus, $I_X R_{in} = V_X - \beta A_0 I_X R_{in}$, and $\dfrac{V_X}{I_X} = R_{in}(1+\beta A_0)$

The input impedance increases by $1 + \beta A_0$, bringing the circuit closer to an ideal voltage amplifier.

8.2.2 Current-voltage feedback

❑ Block diagram: series-series feedback



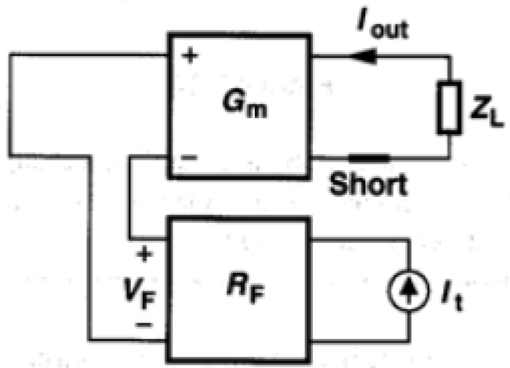
$V_F = R_F I_{out}$, $V_e = V_{in} - R_F I_{out}$, and hence

$I_{out} = G_m (V_{in} - R_F I_{out})$. It follows that

$$\frac{I_{out}}{V_{in}} = \frac{G_m}{1 + G_m R_F}$$

An ideal feedback network in this case exhibits zero input and output impedances.

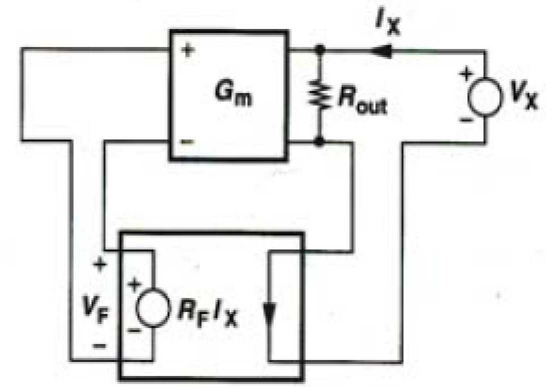
❑ Calculation of loop gain



$V_F = R_F I_t$ and hence $I_{out} = -G_m R_F I_t$.

Thus, the loop gain is equal to $G_m R_F$ and the transconductance of the amplifier is reduced by $1 + G_m R_F$ when feedback is applied.

❑ Calculation of output resistance

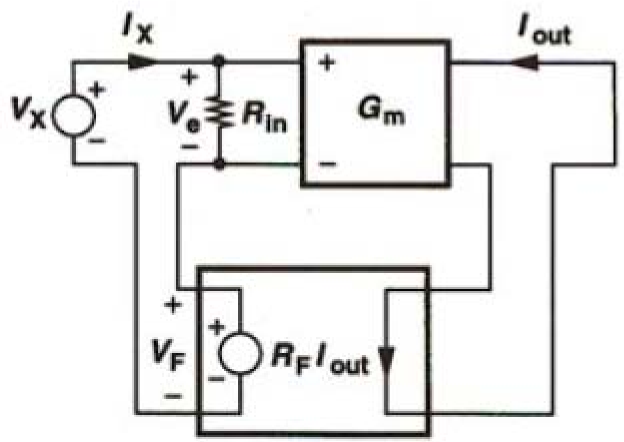


$V_F = R_F I_X$ and $-R_F I_X G_m = I_X - V_X / R_{out}$.

Thus, $$\frac{V_X}{I_X} = R_{out}(1 + G_m R_F)$$

The output impedance increases by $1 + G_m R_F$.

❑ Calculation of input resistance



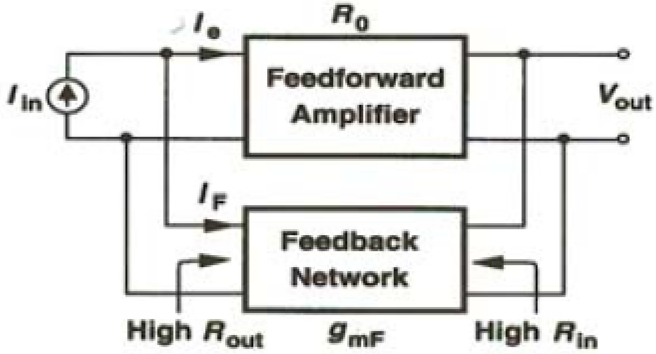
$I_X R_{in} G_m = I_{out}$ and $V_e = V_X - G_m R_F I_X R_{in}$.

Thus, $$\frac{V_X}{I_X} = R_{in}(1 + G_m R_F)$$

The input impedance increases by $1 + G_m R_F$.

8.2.3 Voltage-current feedback

- Block diagram: shunt-shunt feedback



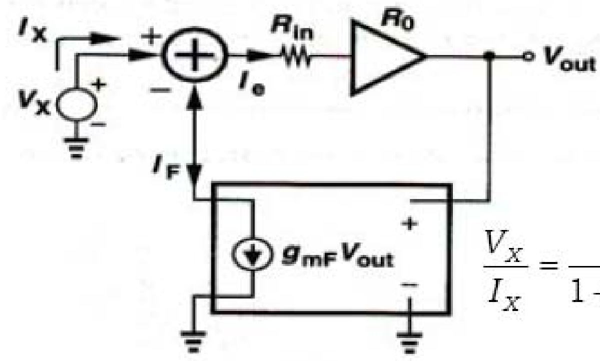
Since $I_F = g_{mF}V_{out}$ and $I_e = I_{in} - I_F$, we have

$V_{out} = R_0 I_e = R_0(I_{in} - g_{mF}V_{out})$. It follows that

$$\frac{V_{out}}{I_{in}} = \frac{R_0}{1 + g_{mF}R_0}$$

Thus, the loop gain is equal to $g_{mF}R_0$ and the transimpedance of the amplifier is reduced by $1 + g_{mF}R_0$ when feedback is applied.

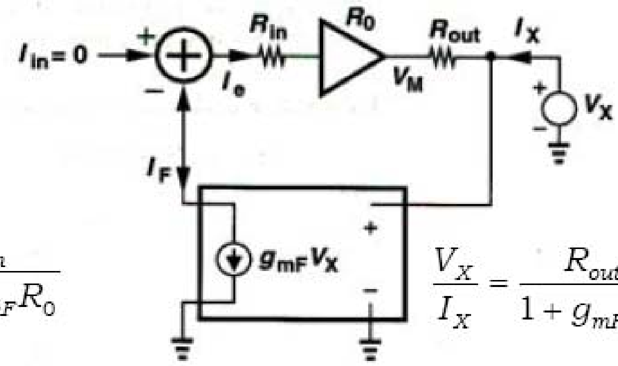
- Calculation of (a) input and (b) output impedances



$$\frac{V_X}{I_X} = \frac{R_{in}}{1 + g_{mF}R_0}$$

(a)

The input impedance is lowered by $1 + g_{mF}R_0$.



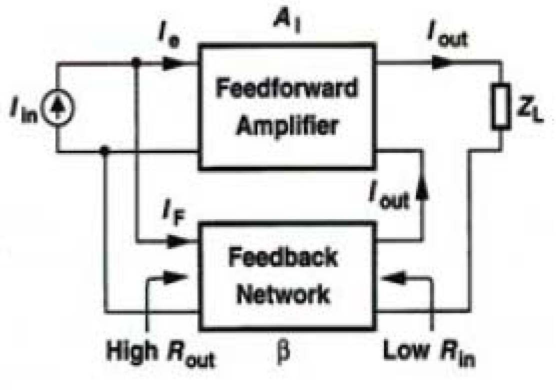
$$\frac{V_X}{I_X} = \frac{R_{out}}{1 + g_{mF}R_0}$$

(b)

The output impedance is lowered by $1 + g_{mF}R_0$.

8.2.4 Current-current feedback

- Block diagram: shut-series feedback

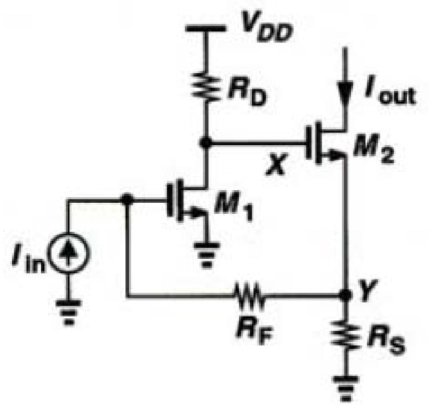


The close-loop gain: $\dfrac{I_{out}}{I_{in}} = \dfrac{A_I}{1+\beta A_I}$

Thus, the loop gain is equal to βA_I and the current gain of the amplifier is reduced by $(1 + \beta A_I)$ when feedback is applied.

Also, the input impedance is divided by $(1 + \beta A_I)$ and the output impedance is multiplied by $(1 + \beta A_I)$.

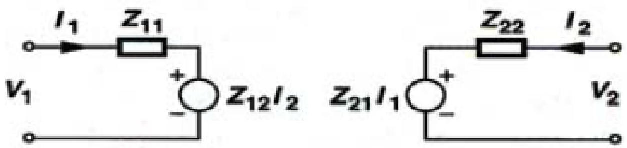
- Example



8.3 Effect of Loading

❑ Two-port network models

- Z model

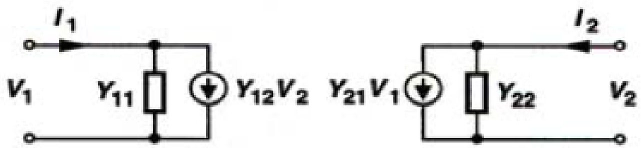


(a)

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

use for *current-voltage* feedback

- Y model

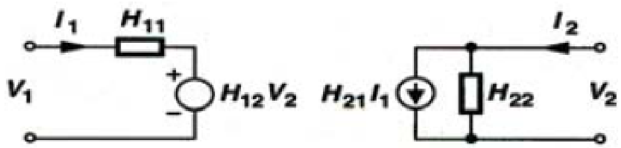


(b)

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

use for *voltage-current* feedback

- H model

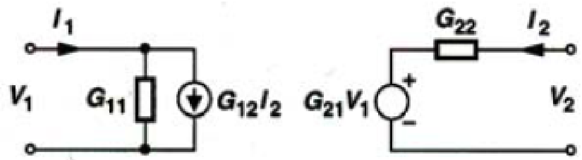


(c)

$$V_1 = H_{11}I_1 + H_{12}V_2$$
$$I_2 = H_{21}I_1 + H_{22}V_2$$

use for *current- current* feedback

- G model



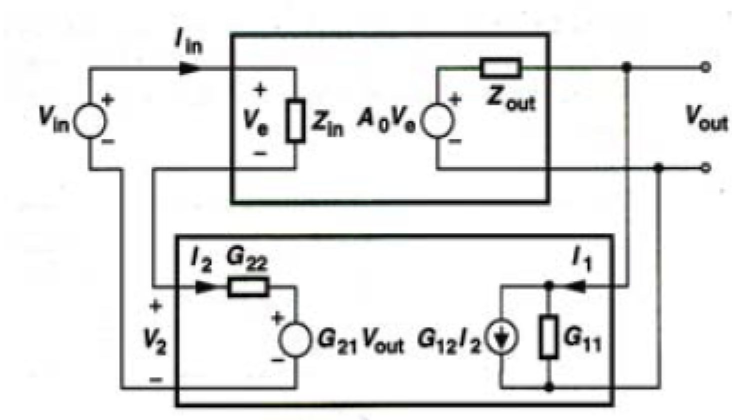
(d)

$$I_1 = G_{11}V_1 + G_{12}I_2$$
$$V_2 = G_{21}V_1 + G_{22}I_2$$

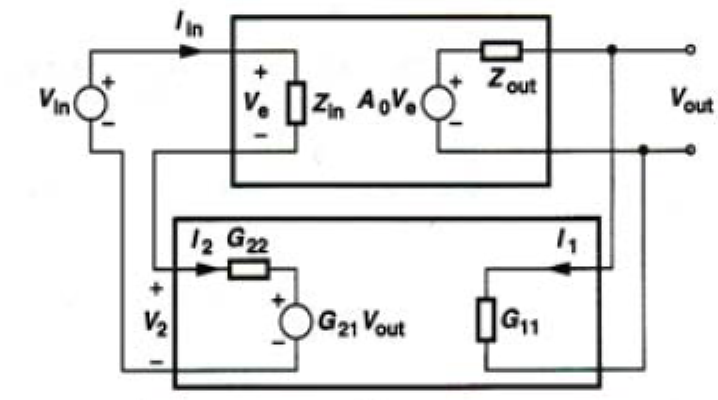
use for *voltage- voltage* feedback

❑ Loading in voltage-voltage feedback

- Block diagram: feedback network represented by G model



(a)



(b) Neglect $G_{12}I_2$

- ❑ If $G_{12} << A_0Z_{in}/Z_{out}$, then the reverse transmission through the feedback circuit is negligible. ($G_{12} \approx 0$)
- ❑ Compute the close-loop gain by (b):

$$V_e = (V_{in} - G_{21}V_{out})\frac{Z_{in}}{Z_{in} + G_{22}}$$

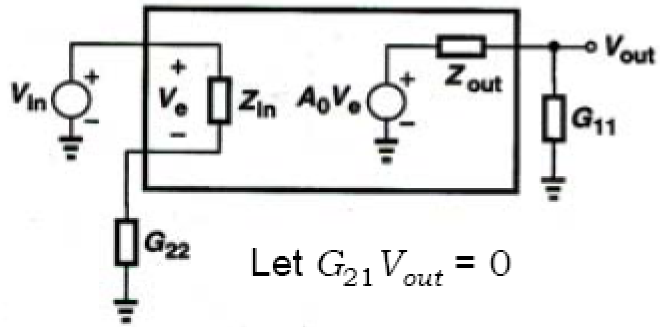
and $(V_{in} - G_{21}V_{out})\dfrac{Z_{in}}{Z_{in} + G_{22}}A_0\dfrac{G_{11}^{-1}}{G_{11}^{-1} + Z_{out}} = V_{out}$

We have $$\frac{V_{out}}{V_{in}} = \frac{A_0\dfrac{Z_{in}}{Z_{in} + G_{22}} \cdot \dfrac{G_{11}^{-1}}{G_{11}^{-1} + Z_{out}}}{1 + \dfrac{Z_{in}}{Z_{in} + G_{22}} \cdot \dfrac{G_{11}^{-1}}{G_{11}^{-1} + Z_{out}}G_{21}A_0}$$

- ❑ If the feedback network is ideal, i.e, $G_{11}{}^{-1} = \infty$, and $G_{22} = 0$, then

$$\frac{V_{out}}{V_{in}} = \frac{A_0}{1 + G_{21}A_0}$$

- The loaded open loop gain

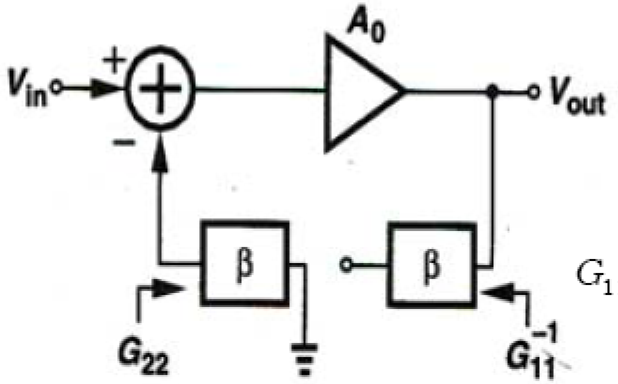


Let $G_{21} V_{out}$ = 0

By , $\frac{V_{out}}{V_{in}} = \frac{A_0 \frac{Z_{in}}{Z_{in}+G_{22}} \cdot \frac{G_{11}^{-1}}{G_{11}^{-1}+Z_{out}}}{1+\frac{Z_{in}}{Z_{in}+G_{22}} \cdot \frac{G_{11}^{-1}}{G_{11}^{-1}+Z_{out}} G_{21} A_0} = \frac{A_{v,open}}{1+G_{21}A_{v,open}}$

we have $A_{v,open} = A_0 \frac{Z_{in}}{Z_{in}+G_{22}} \cdot \frac{G_{11}^{-1}}{G_{11}^{-1}+Z_{out}}$.

- Conceptual view



□ The loop gain is equal to $G_{21}A_{v,open}$.

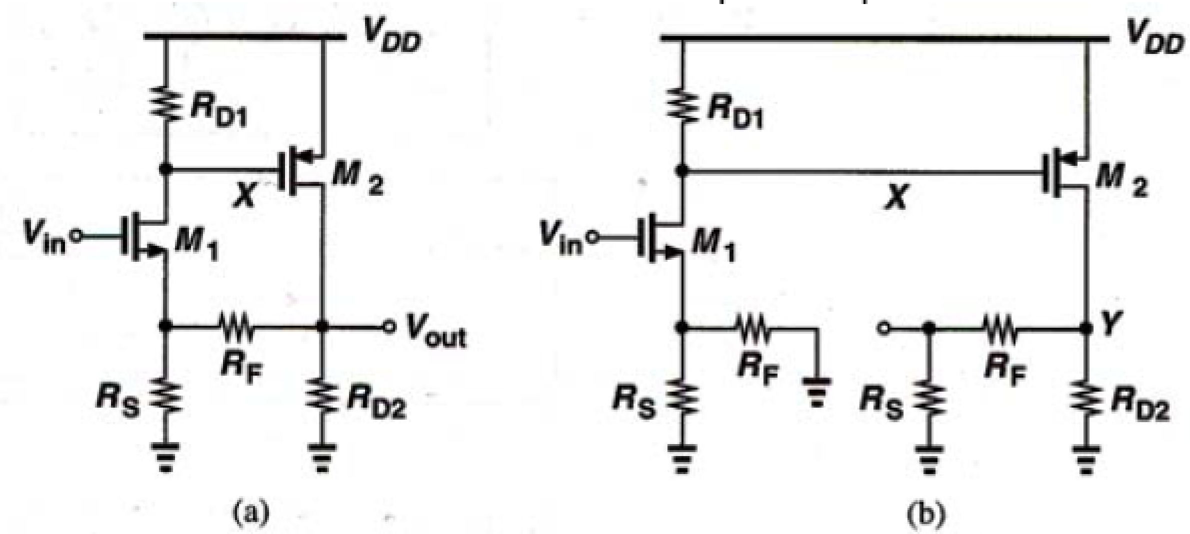
□ The open-loop input and output impedance are scaled by $1 + G_{21}A_{v,open}$ to yield the close-loop values.

$$G_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0}$$

$$G_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0}$$

- Example

Open-loop circuit



Neglecting channel-length modulation and body effect, we have

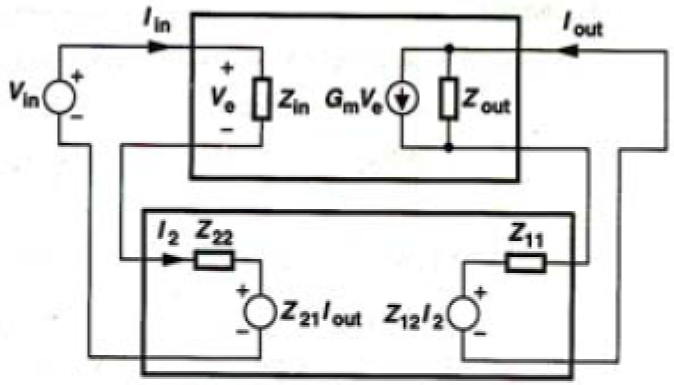
$$A_{v,open} = \frac{V_Y}{V_{in}} = \frac{-R_{D1}}{R_F \| R_S + 1/g_{m1}} \{-g_{m2}[R_{D2} \| (R_F + R_S)]\}$$

Compute the close loop gain:

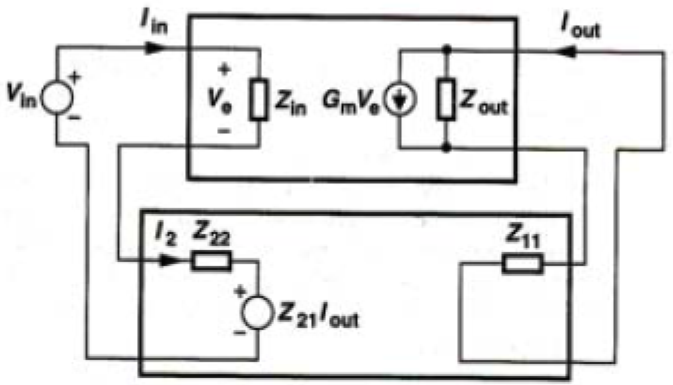
$$G_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0} = \frac{R_S}{R_F + R_S}, \text{ and } \quad A_{v,closed} = \frac{A_{v,open}}{1 + G_{21}A_{v,open}}$$

❑ Loading in current-voltage feedback

- Block diagram: feedback network represented by Z model



(a)



(b)

Compute the close-loop gain by (b):

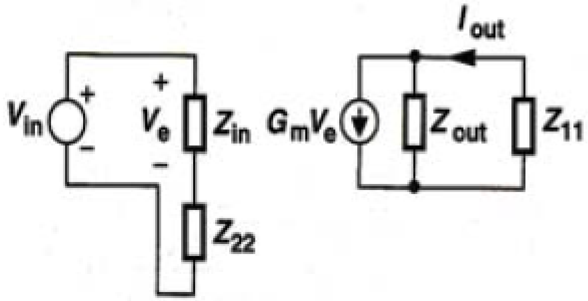
$$(V_{in} - Z_{21}I_{out})\frac{Z_{in}}{Z_{in}+Z_{22}}G_m\frac{Z_{out}}{Z_{out}+Z_{11}} = I_{out}$$

We have $$\frac{I_{out}}{V_{in}} = \frac{\frac{Z_{in}}{Z_{in}+Z_{22}}\cdot\frac{Z_{out}}{Z_{out}+Z_{11}}G_m}{1+\frac{Z_{in}}{Z_{in}+Z_{22}}\cdot\frac{Z_{out}}{Z_{out}+Z_{11}}G_mZ_{21}}$$

The loaded open-loop gain is equal to

$$G_{m,open} = \frac{Z_{in}}{Z_{in}+Z_{22}}\cdot\frac{Z_{out}}{Z_{out}+Z_{11}}G_m$$

- The loaded open-loop gain

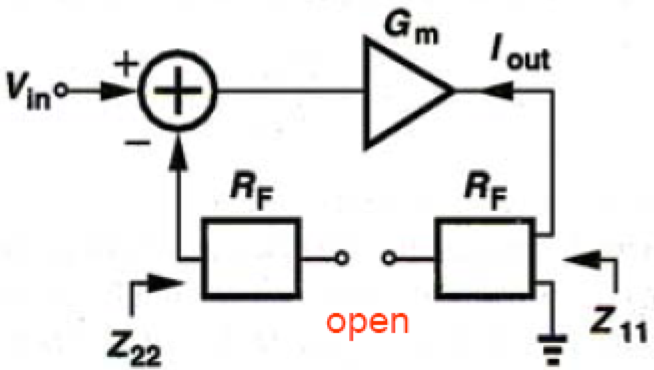


The loaded open-loop gain is equal to

$$G_{m,open} = \frac{Z_{in}}{Z_{in} + Z_{22}} \cdot \frac{Z_{out}}{Z_{out} + Z_{11}} G_m$$

revealing voltage division at input and current division at the output.

- Conceptual view

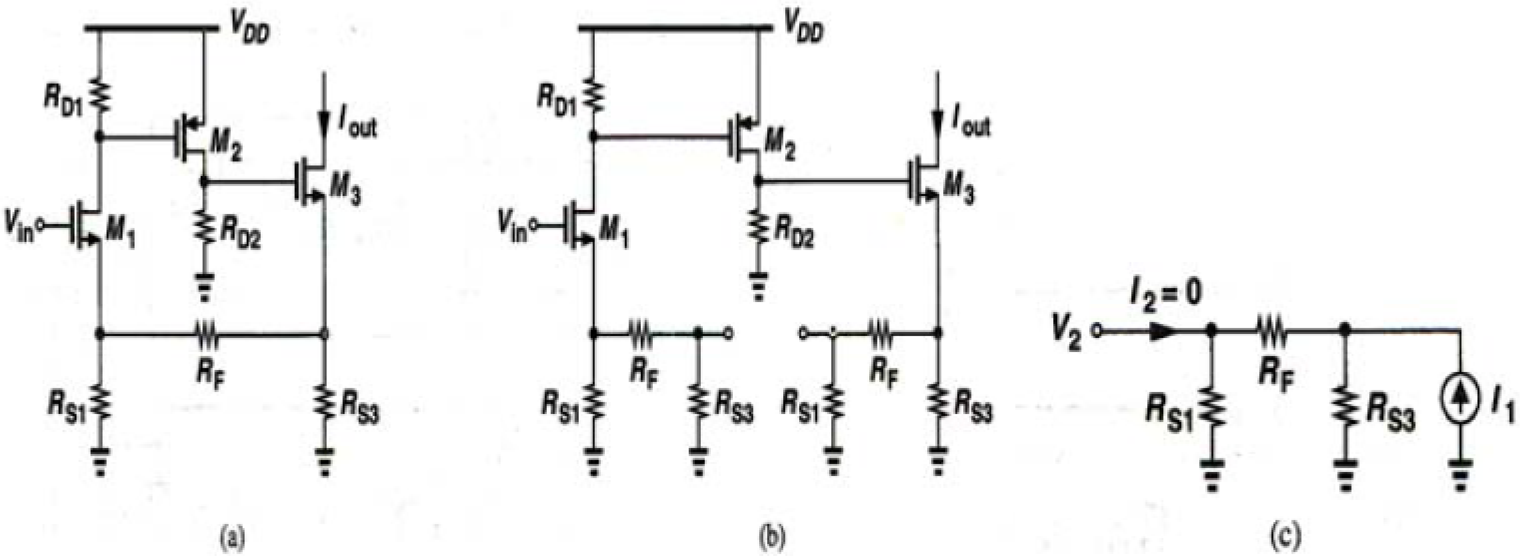


$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

- Example

Open-loop circuit



Neglecting channel-length modulation and body effect, we have

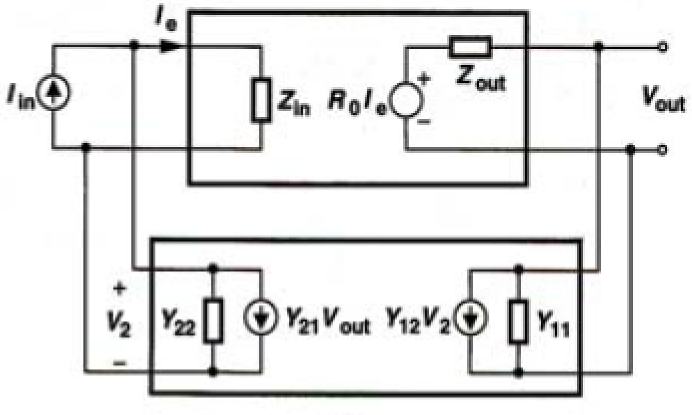
$$G_{m,open} = \frac{-R_{D1}}{R_{S1} \| (R_F + R_{S3}) + 1/g_{m1}} \cdot \frac{-g_{m2}R_{D2}}{R_{S3} \| (R_F + R_{S1}) + 1/g_{m3}}$$

Compute the close loop gain:

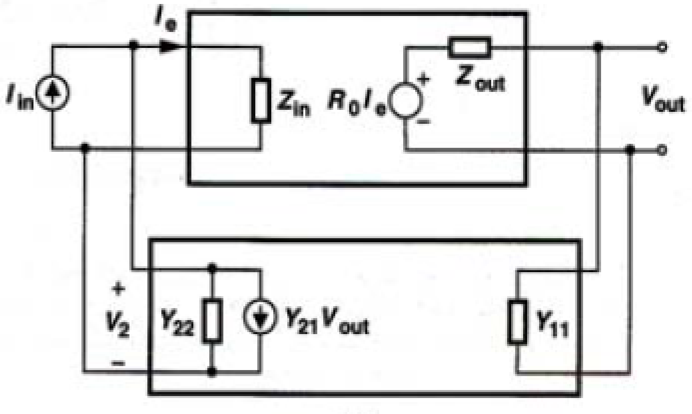
By (c), $Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{R_{S3}}{R_{S3} + R_{S1} + R_F} R_{S1}$. Thus, $G_{m,closed} = \frac{G_{m,open}}{1 + Z_{21}G_{m,open}}$

❑ Loading in voltage-current feedback

- Block diagram: feedback network represented by Y model



(a)



(b)

Compute the close-loop gain by (b):

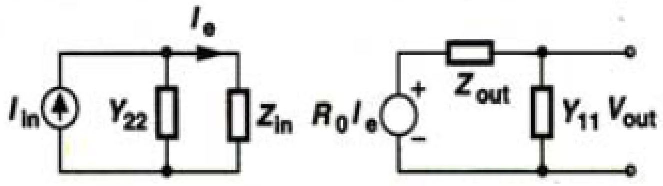
$$(I_{in} - Y_{21}V_{out})\frac{Y_{22}^{-1}}{Y_{22}^{-1} + Z_{in}} R_0 \frac{Y_{11}^{-1}}{Y_{11}^{-1} + Z_{out}} = V_{out}$$

We have
$$\frac{V_{out}}{I_{in}} = \frac{\frac{Y_{22}^{-1}}{Y_{22}^{-1} + Z_{in}} R_0 \frac{Y_{11}^{-1}}{Y_{11}^{-1} + Z_{out}}}{1 + \frac{Y_{22}^{-1}}{Y_{22}^{-1} + Z_{in}} R_0 \frac{Y_{11}^{-1}}{Y_{11}^{-1} + Z_{out}} Y_{21}}$$

The loaded open-loop gain is equal to

$$R_{0,open} = \frac{Y_{22}^{-1}}{Y_{22}^{-1} + Z_{in}} \cdot \frac{Y_{11}^{-1}}{Y_{11}^{-1} + Z_{out}} R_0$$

- The loaded open-loop gain

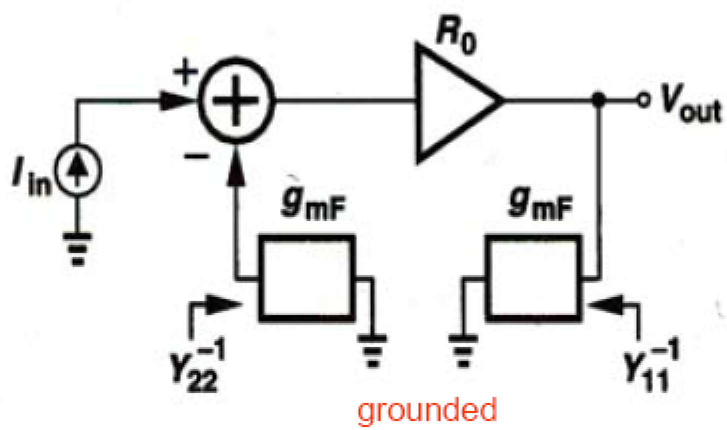


The loaded open-loop gain is equal to

$$R_{0,open} = \frac{Y_{22}^{-1}}{Y_{22}^{-1} + Z_{in}} \cdot \frac{Y_{11}^{-1}}{Y_{11}^{-1} + Z_{out}} R_0$$

revealing current division between $Y_{22}{}^{-1}$ and Z_{in} and voltage division between Z_{out} and $Y_{11}{}^{-1}$.

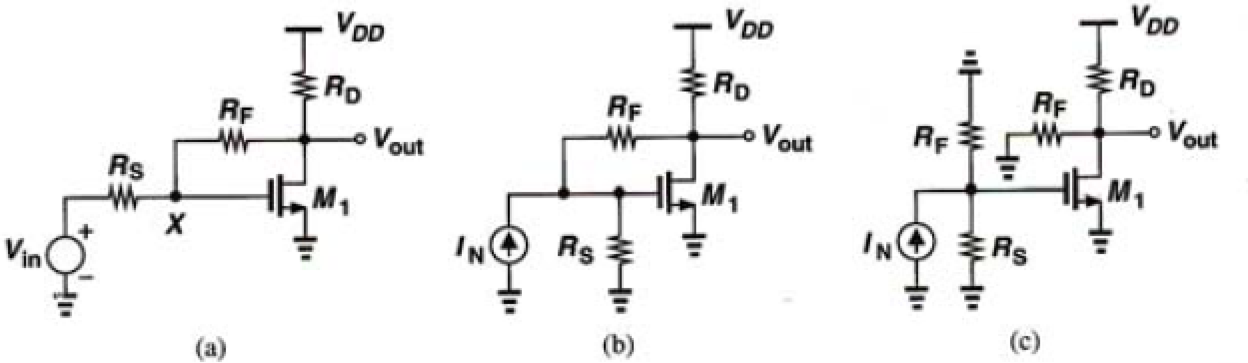
- Conceptual view



$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

- Example



Neglecting channel-length modulation and body effect, we have

$$R_{0,open} = \left.\frac{V_{out}}{I_N}\right|_{open} = -(R_S \| R_F) g_m (R_F \| R_D)$$

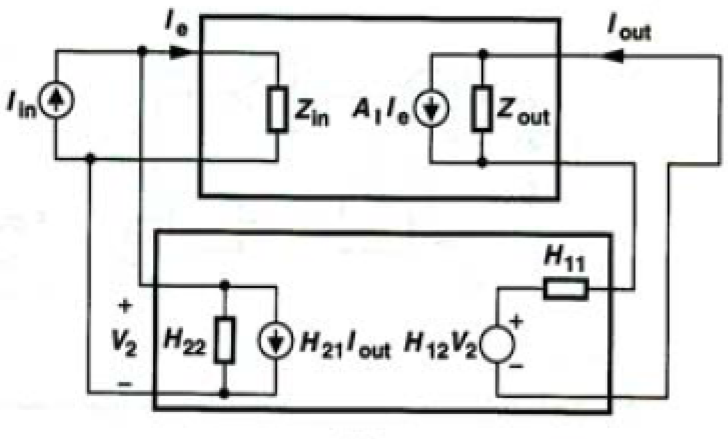
where $I_N = V_{in}/R_S$. The loop gain is equal to $Y_{21}R_{0,open}$.

Compute the close loop gain:

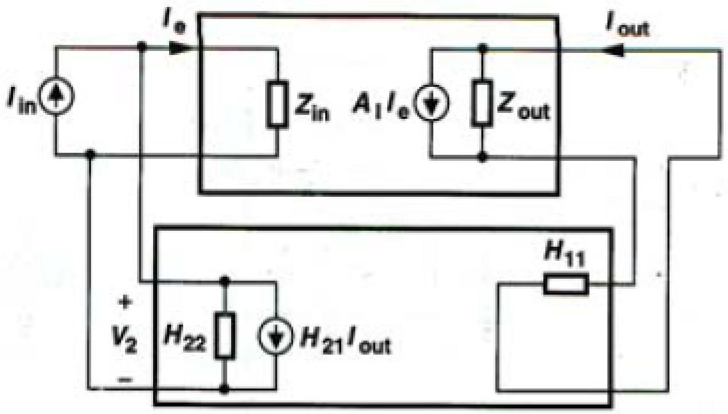
$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -\frac{1}{R_F}\text{ . Thus, } R_{0,closed} = \frac{R_{0,open}}{1+Y_{21}R_{0,open}} \text{ and } \frac{V_{out}}{V_{in}} = \frac{V_{out}}{I_N R_S} = \frac{1}{R_S}\left.\frac{V_{out}}{I_N}\right|_{close}$$

❑ Loading in current-current feedback

- Block diagram: feedback network represented by H model



(a)



(b)

Compute the close-loop gain by (b):

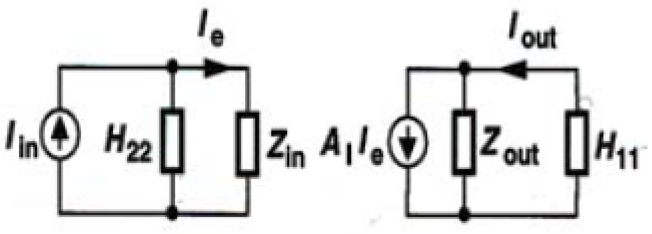
$$\left(I_{in} - H_{21} I_{out}\right) \frac{H_{22}^{-1}}{H_{22}^{-1} + Z_{in}} A_I \frac{Z_{out}}{H_{11} + Z_{out}} = I_{out}$$

We have $\frac{I_{out}}{I_{in}} = \dfrac{\dfrac{H_{22}^{-1}}{H_{22}^{-1} + Z_{in}} A_I \dfrac{Z_{out}}{H_{11} + Z_{out}}}{1 + \dfrac{H_{22}^{-1}}{H_{22}^{-1} + Z_{in}} A_I \dfrac{Z_{out}}{H_{11} + Z_{out}} H_{21}}$

The loaded open-loop gain is equal to

$$A_{I,open} = \frac{H_{22}^{-1}}{H_{22}^{-1} + Z_{in}} \cdot \frac{Z_{out}}{H_{11} + Z_{out}} A_I$$

- The loaded open-loop gain

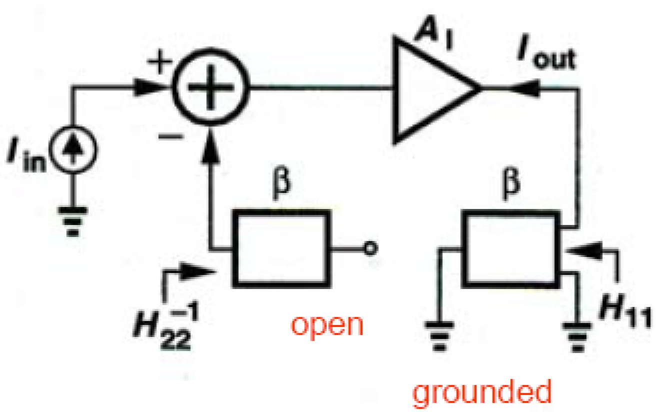


The loaded open-loop gain is equal to

$$A_{I,open} = \frac{H_{22}^{-1}}{H_{22}^{-1} + Z_{in}} \cdot \frac{Z_{out}}{H_{11} + Z_{out}} A_I$$

revealing that the feedback network introduces division at the both input and output of the system.

- Conceptual view

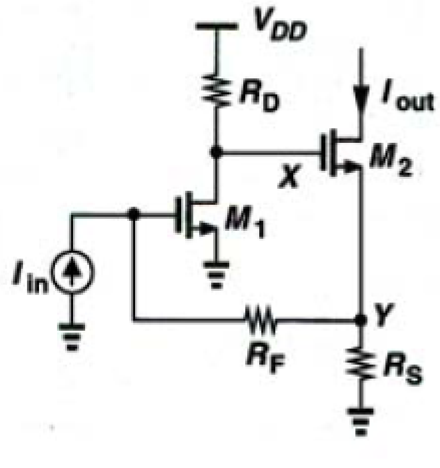


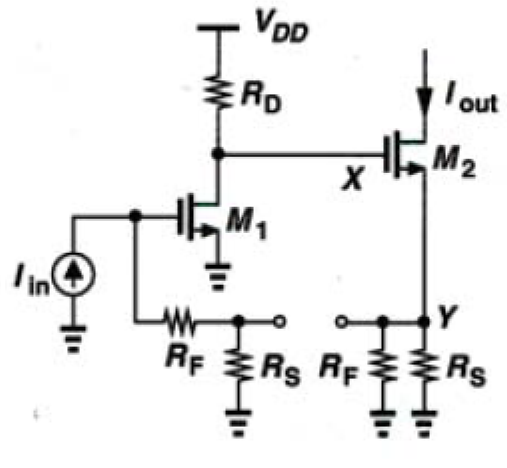
$$H_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$$

$$H_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

- Example

Open-loop circuit





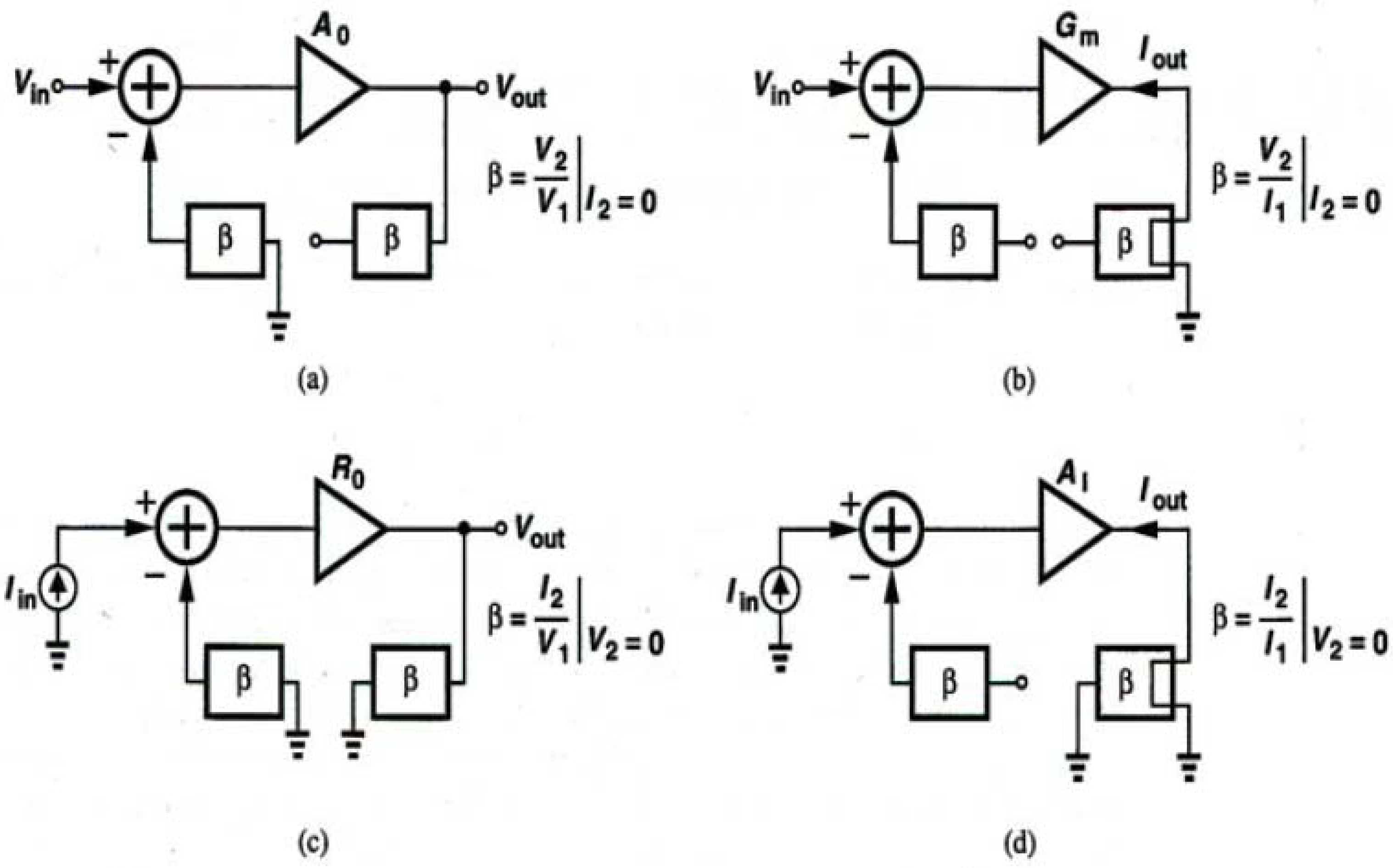
Neglecting channel-length modulation and body effect, we have

$$A_{I,open} = -(R_F + R_S) g_{m1} R_D \frac{1}{R_S \| R_F + 1/g_{m2}}$$

Compute the close loop gain:

$$H_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = -\frac{R_S}{R_S + R_F}$$, and $$A_{I,closed} = \frac{A_{I,open}}{1 + H_{21} A_{I,open}}$$

❑ Summary of loading effects



(a) (b) (c) (d)