Design of Analog CMOS Integrated Circuits

<Chapter 4>

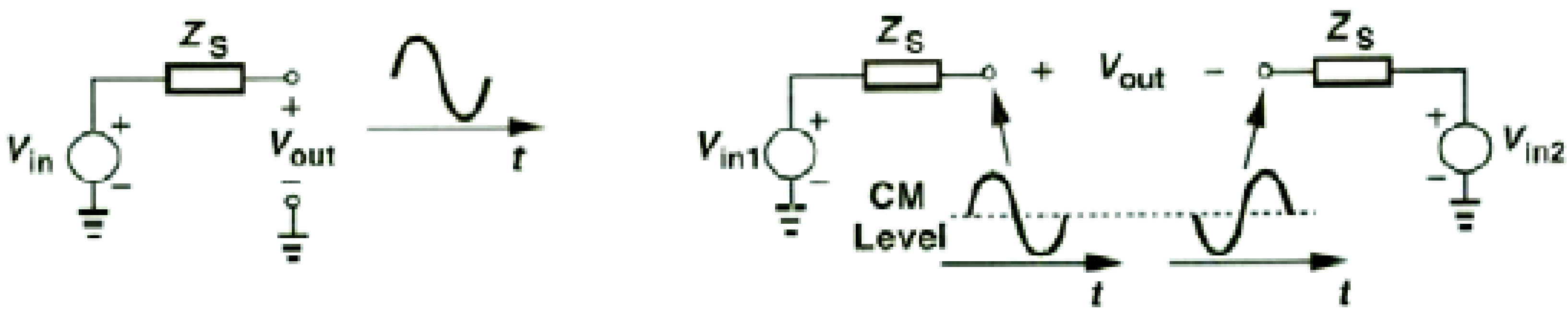
Differential Amplifiers

양병도

4.1 Single-ended vs. Differential Signals

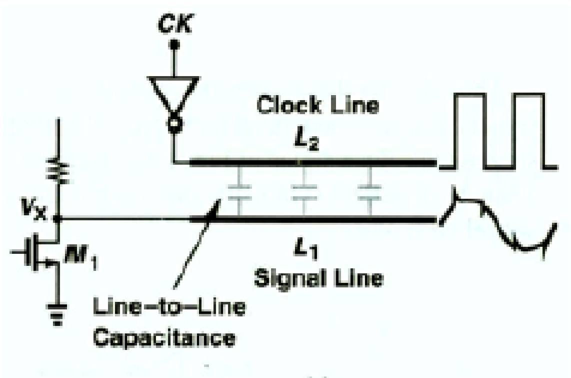
❑ Differential Signals

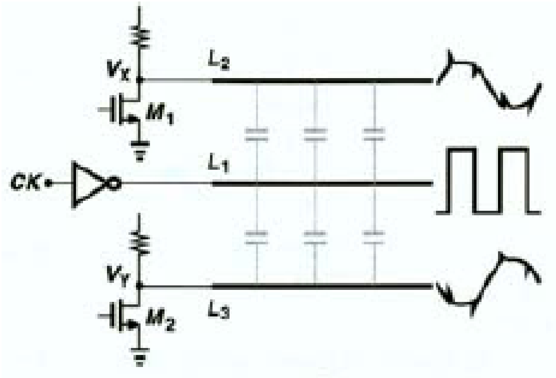
- ✓ 장점:
  - ➢ 잡음에 강하다 → Common mode (CM) noise 제거
  - ➢ Swing 전압 증가 → $2\times[V_{DD}-(V_{GS}-V_{TH})]$
  - ➢ Bias 쉽고 linearity 증가
- ✓ 단점:
  - ➢ 면적/ 전력 증가



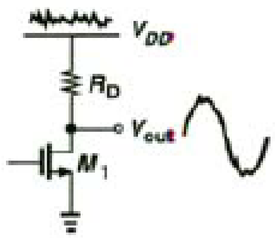
Advantages of differential operation

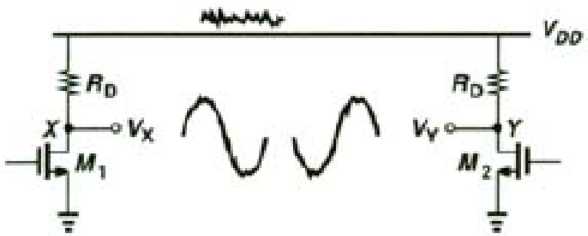
- Reduction of coupling





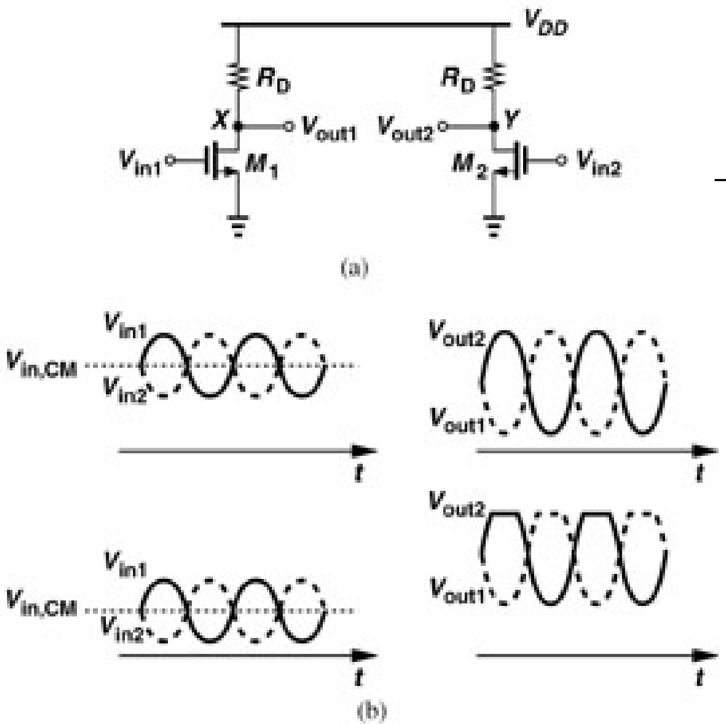
- Common-mode rejection occurs with noisy supply voltages

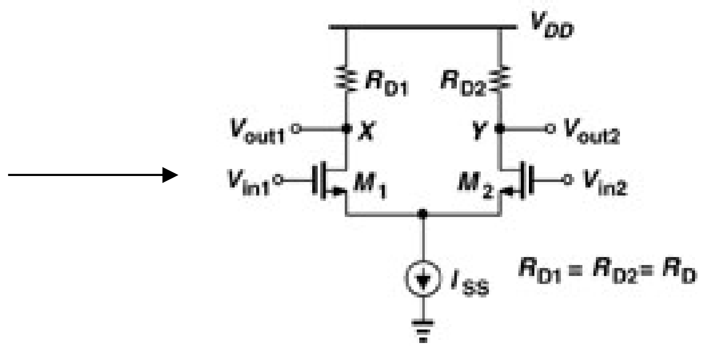




4.2 Basic differential pair

- 문제점: v_{in1}과 v_{in2}의 CM level 변화 → bias current 변화 → gain 변화
- 해결: bias current source 사용 → CM level 변화 → bias current 일정 → gain 일정





If $V_{in1}=V_{in2}$ then $I_{D1}=I_{D2}=I_{SS}/2$

Then $V_{out1}=V_{out2}=V_{DD}-R_DI_{SS}/2$

정성적 분석

If $V_{in1}>>V_{in2}$,

M1 off → $V_{out1}=V_{DD}$

M2 on → I_{SS} → $V_{out2}=V_{DD}-R_DI_{SS}$

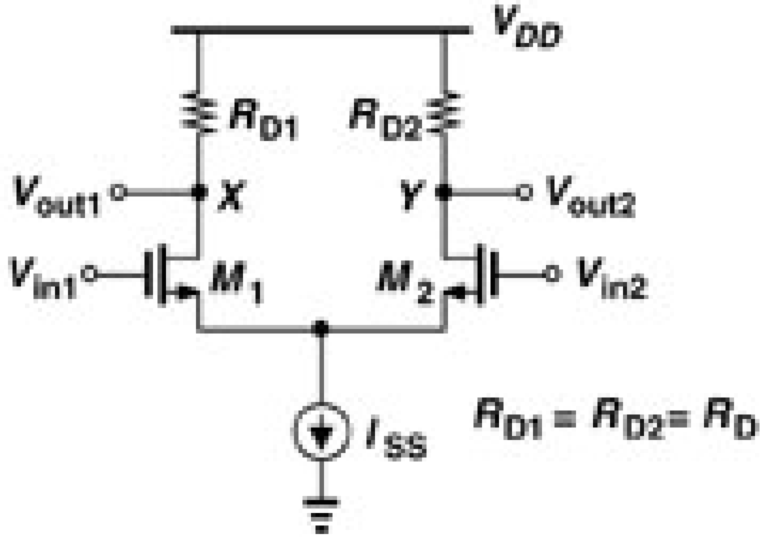
If Vin1=Vin2

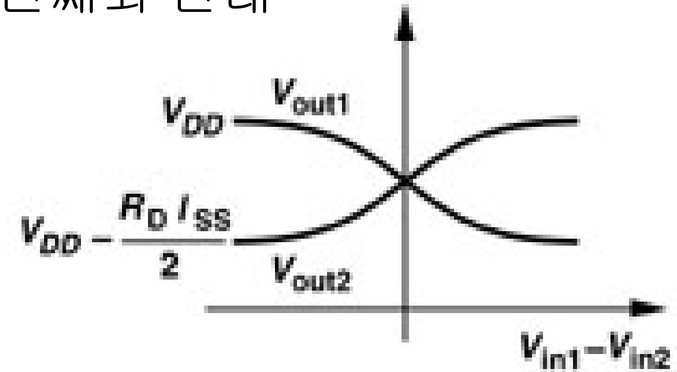
$I_{D1}=I_{D2}=I_{SS}/2$

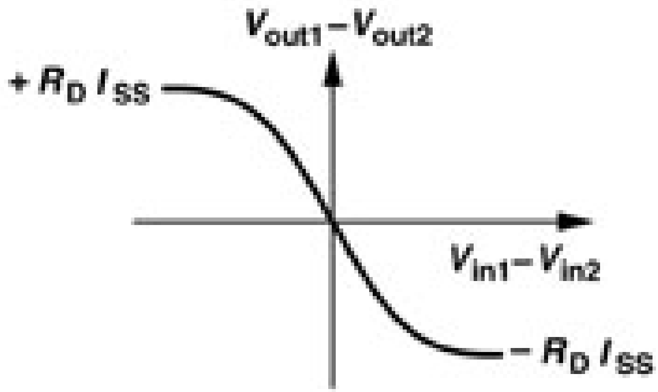
$V_{out1}=V_{out2}=V_{DD}-R_DI_{SS}/2$

If $V_{in1}<<V_{in2}$,

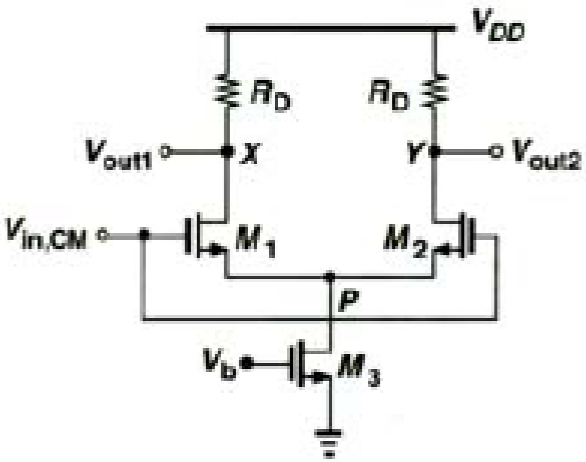
첫 번째와 반대







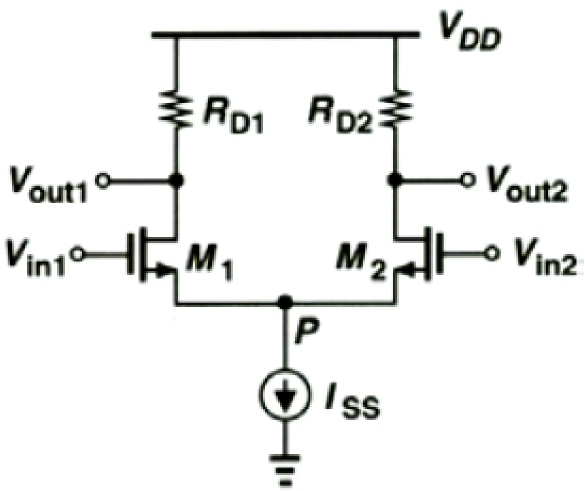
Common-Mode Response



- M3가 saturation에서 동작
  - ✓ → $V_p > V_{GS3} - V_{TH3}$
  - ✓ → $V_{in,CM} > V_{GS1} + (V_{GS3} - V_{TH3})$
- M1이 saturation에서 동작
  - ✓ → $V_{out1} > V_{in,CM} - V_{TH1}$
  - ✓ → $V_{in,CM} < V_{out1} + V_{TH1}$
  - ✓ $= V_{DD} - R_D I_{SS}/2 + V_{TH1}$

$$V_{GS1} + (V_{GS3} - V_{TH3}) \le V_{in,CM} \le \min[V_{DD} - R_D \frac{I_{SS}}{2} + V_{TH1}, V_{DD}]$$

정량적 분석(1/3)



$V_{out1} = V_{DD} - R_{D1}I_{D1}$, $V_{out2} = V_{DD} - R_{D2}I_{D2}$

$[R_{D1} = R_{D2} = R_D]$

⇨ $V_{out1} - V_{out2} = R_{D2}I_{D2} - R_{D1}I_{D1} = R_D\,(I_{D2} - I_{D1})$

Assuming the circuit is symmetric, M_1 and M_2 are saturated, and $\lambda = 0$,

$V_{in1} - V_{in2} = V_{GS1} - V_{GS2}$

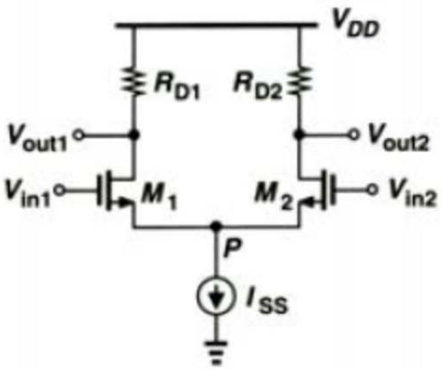
For a square-law device, we have:

Therefore,

$$V_{in1} - V_{in2} = \sqrt{\frac{2I_{D1}}{\mu_n C_{ox}\frac{W}{L}}} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox}\frac{W}{L}}}$$

Recognizing that $I_{D2} + I_{D1} = I_{SS}$, we obtain

정량적 분석(2/3)



$$\Rightarrow \quad (V_{in1}-V_{in2})^2 = \frac{2}{\mu_n C_{ox}\frac{W}{L}}\left(I_{SS}-2\sqrt{I_{D1}I_{D2}}\right)$$

$$\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in1}-V_{in2})^2 - I_{SS} = -2\sqrt{I_{D1}I_{D2}}$$

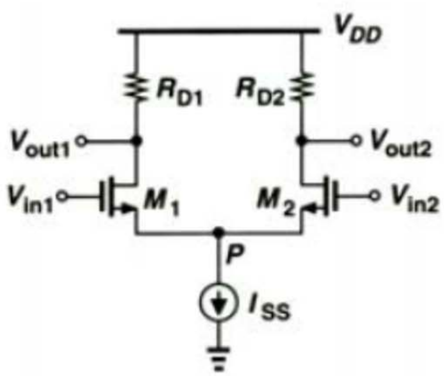
Squaring the two sides again and $4I_{D1}I_{D2} = (I_{D1}+I_{D2})^2 - (I_{D1}-I_{D2})^2 = I_{SS}{}^2 - (I_{D1}-I_{D2})^2$,

we arrive at $I_{D1}-I_{D2} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in1}-V_{in2})\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}}-(V_{in1}-V_{in2})^2}$

Denoting $\Delta I_D = I_{D1} - I_{D2}$ and $\Delta V_{in} = V_{in1} - V_{in2}$, we can show that

$$\frac{\partial \Delta I_D}{\partial \Delta V_{in}} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\frac{\frac{4I_{SS}}{\mu_n C_{ox}W/L}-2\Delta V_{in}^2}{\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}W/L}-\Delta V_{in}^2}}$$

정량적 분석(3/3)

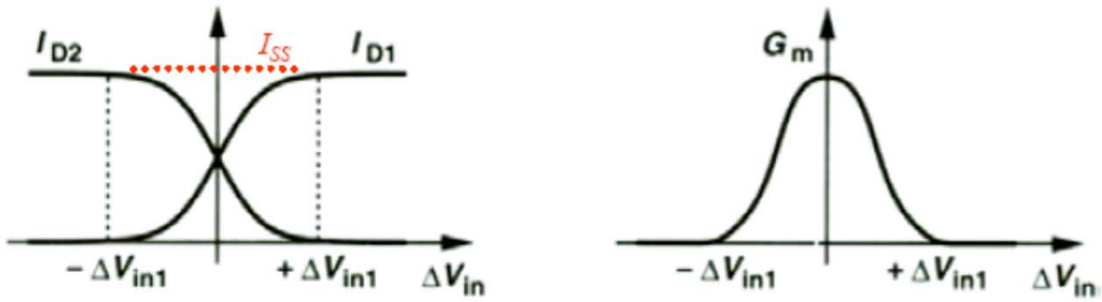


- For $\Delta V_{in} = 0$, $G_m = \left.\dfrac{\partial \Delta I_D}{\partial \Delta V_{in}}\right|_{V_{in}=0} = \sqrt{\mu_n C_{ox} \dfrac{W}{L} I_{SS}}$
- Since $\Delta V_{out} = V_{out1} - V_{out2} = R_D \Delta I_D = R_D G_M \Delta V_{in}$, the small-signal differential gain is given as

$$|A_v| = G_m R_D = \sqrt{\mu_n C_{ox} \frac{W}{L} I_{SS}} \cdot R_D$$

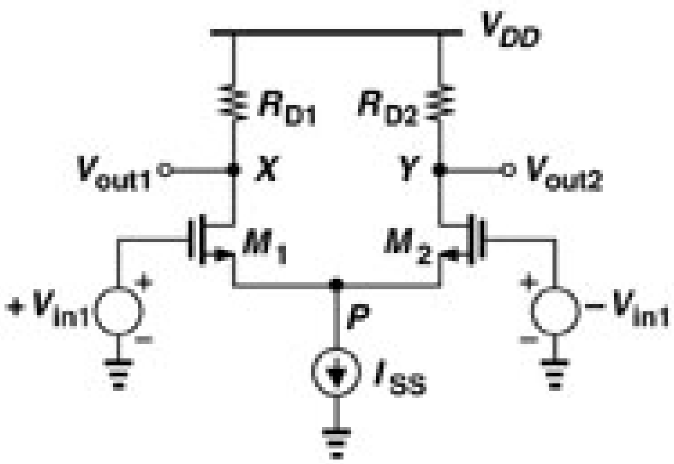
- $G_M = 0$ for $\Delta V_{in} = \sqrt{\dfrac{2 I_{SS}}{\mu_n C_{ox} W / L}}$

Variation of I_D and G_M vs. ΔV_{in}



Small signal 해석 (Half circuit 이용)

- Differential pair가 완전히 symmetric한 경우
- Small signal의 node P를 virtual ground로 본다.
- CS와 같은 방식으로 해석 → 간단해짐



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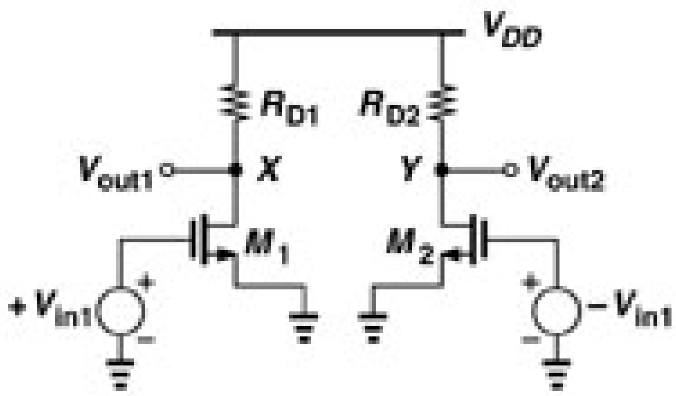
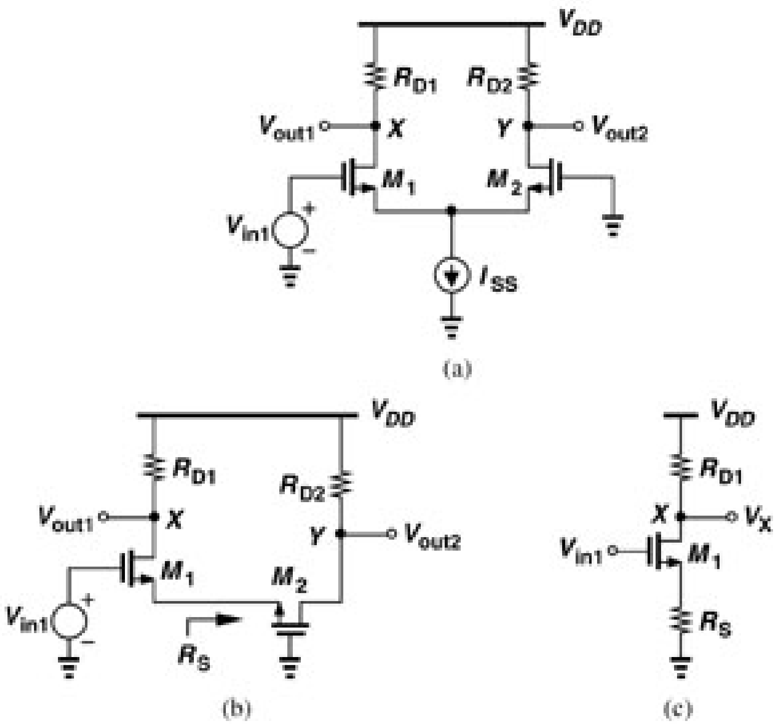


(b)

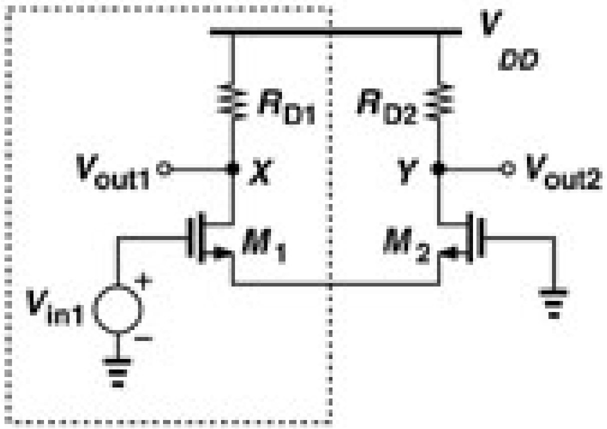
Small signal 해석 (superposition 이용)

- (1) V_{in2}=0 일때
  - ✓ Neglecting λ_2 and g_{mb2} $R_S \approx 1/g_{m2}$
- Differential Gain: Effect of V_{in1} on V_X

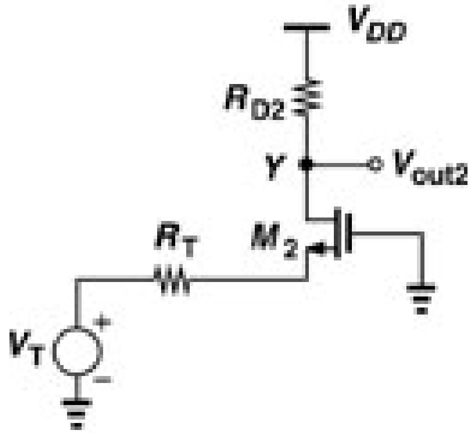


$$\frac{V_X}{V_{in1}} \approx \frac{-R_D}{\dfrac{1}{g_{m1}} + \dfrac{1}{g_{m2}}}$$

❑ Differential Gain: Effect of V_{in1} on V_Y



(a)



(b)

$$\frac{V_Y}{V_{in1}} \approx \frac{R_D}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}}$$

❑ V_X-V_Y as function of V_{in1} if $V_{in2}=0$

- ✓ $g_{m1}=g_{m2}=g_m$

$$(V_X - V_Y)_{due\_to\_V_{in1}} = \frac{-2R_D}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}} V_{in1} = -g_m R_D V_{in1}$$

- (2) V_{in1}=0 일 때
- V_X-V_Y as function of V_{in2} if V_{in1}=0

$$(V_X - V_Y)_{due\_to\_V_{in2}} = \frac{2R_D}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}} V_{in2} = g_m R_D V_{in2}$$

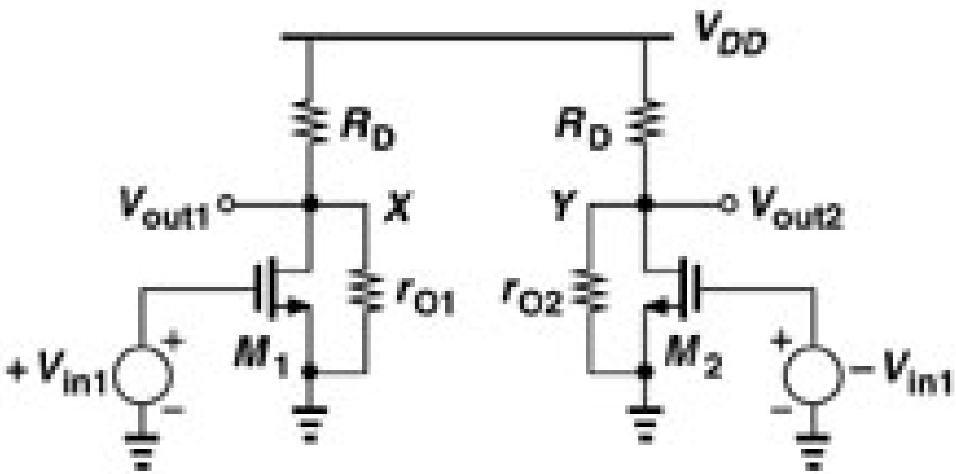
- V_X-V_Y as function of V_{in2} and V_{in1}

$$(V_X - V_Y)_{due\_to\_both} = g_m R_D V_{in2} - g_m R_D V_{in1}$$

$$A_v(diff) = \frac{V_{out1} - V_{out2}}{V_{in1} - V_{in2}} = g_m R_D$$

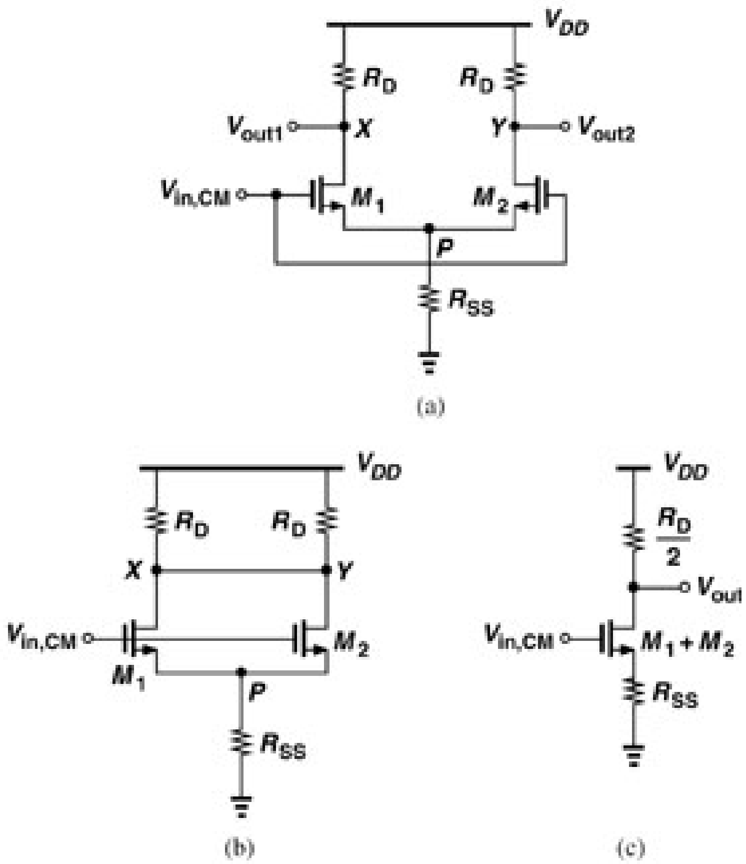
$$A_v(S.E.) = \frac{g_m}{2} R_D$$

Differential Gain with λ effect



- Based on the half-circuit concept, gain calculation is highly simplified.
- $V_X/V_{in1}=-g_m(R_D||r_o)=V_Y/(-V_{in1})$
- $(V_X-V_Y)/2V_{in1}=-g_m(R_D||r_o)$

4.3 Common mode response

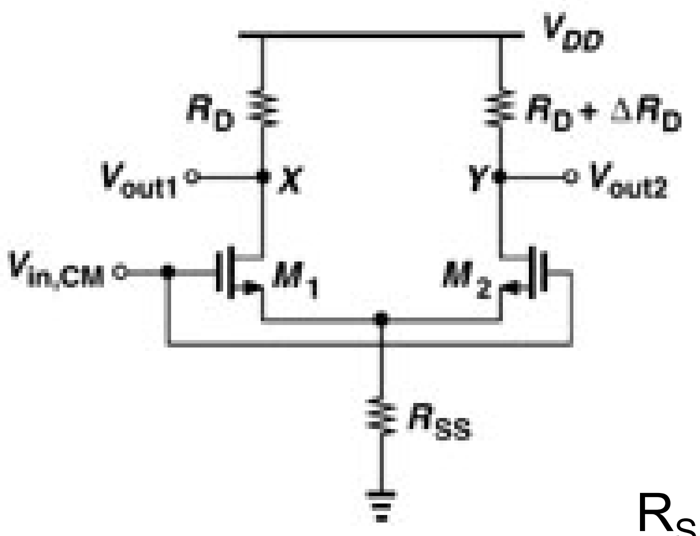


$$A_{V,CM} = \frac{V_{out}}{V_{in,CM}} = \frac{V_X}{V_{in,CM}} = \frac{V_Y}{V_{in,CM}}$$

$$= -\frac{R_D/2}{1/(2g_m) + R_{SS}}$$

M_1+M_2 has twice the width and bias current, therefore g_m is doubled.

Common-Mode Response with asymmetric R_D assuming λ=0



$$\frac{\Delta V_X}{\Delta V_{in,CM}} = -\frac{g_m}{1+2g_m R_{SS}} R_D$$

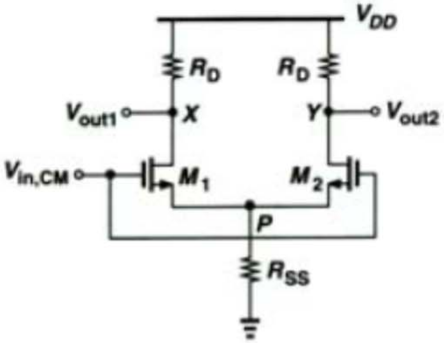
$$\frac{\Delta V_Y}{\Delta V_{in,CM}} = -\frac{g_m}{1+2g_m R_{SS}} (R_D + \Delta R_D)$$

$$\frac{\Delta V_X - \Delta V_Y}{\Delta V_{in,CM}} = \frac{g_m}{1+2g_m R_{SS}} \Delta R_D$$

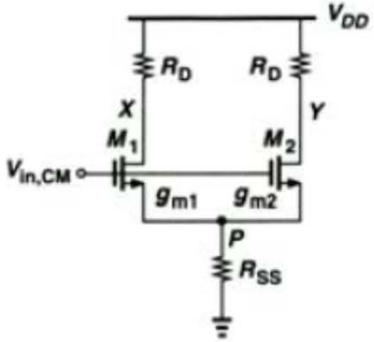
R_{SS} *above represents the current source*

→ *need large* R_{SS}

g_m mismatch



(a)



(b)

$$\begin{cases} I_{D1} = g_{m1}\left(V_{in,CM} - V_P\right) \\ I_{D2} = g_{m2}\left(V_{in,CM} - V_P\right) \end{cases}$$

$$\Rightarrow \quad \left(g_{m1} + g_{m2}\right)\left(V_{in,CM} - V_P\right) = V_P / R_{SS}$$

$$\Rightarrow \quad V_P = \frac{\left(g_{m1} + g_{m2}\right)R_{SS}}{\left(g_{m1} + g_{m2}\right)R_{SS} + 1} V_{in,CM}$$

$$\begin{cases} V_X = -g_{m1}\left(V_{in,CM} - V_P\right)R_D = -\dfrac{g_{m1}}{\left(g_{m1} + g_{m2}\right)R_{SS} + 1} R_D V_{in,CM} \\ V_Y = -g_{m2}\left(V_{in,CM} - V_P\right)R_D = -\dfrac{g_{m2}}{\left(g_{m1} + g_{m2}\right)R_{SS} + 1} R_D V_{in,CM} \end{cases} \Rightarrow \quad V_X - V_Y = -\frac{g_{m1} - g_{m2}}{\left(g_{m1} + g_{m2}\right)R_{SS} + 1} R_D V_{in,CM}$$

Thus, the circuit converts input CM variations to a differential error by a factor equal to ➡

$$A_{CM-DM} = -\frac{\Delta g_m R_D}{\left(g_{m1} + g_{m2}\right)R_{SS} + 1}$$

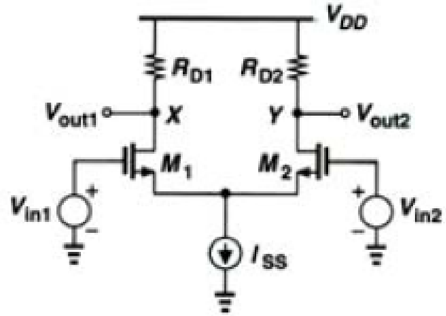
A_{CM-DM} : CM to DM conversion

$\Delta g_m = g_{m1} - g_{m2}$

Common-Mode Rejection Ratio (CMRR)

Definition $CMRR = \left| \dfrac{A_{DM}}{A_{CM-DM}} \right|$

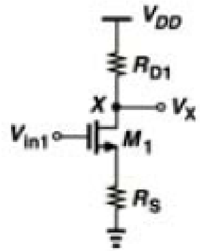
If only g_m mismatch is considered



- Differential mode (assume $V_{in1} = -V_{in2}$)

$$|A_{DM}| = \frac{R_D}{2} \frac{g_{m1} + g_{m2} + 4g_{m1}g_{m2}R_{SS}}{1 + (g_{m1} + g_{m2})R_{SS}}$$

- Common-mode to differential-mode conversion

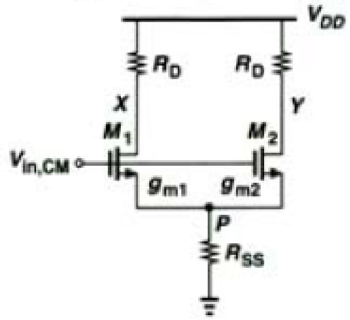


(c)

$$R_S = (1/g_{m2}) \| R_{SS}$$

$$A_{CM-DM} = -\frac{\Delta g_m R_D}{(g_{m1} + g_{m2})R_{SS} + 1}$$

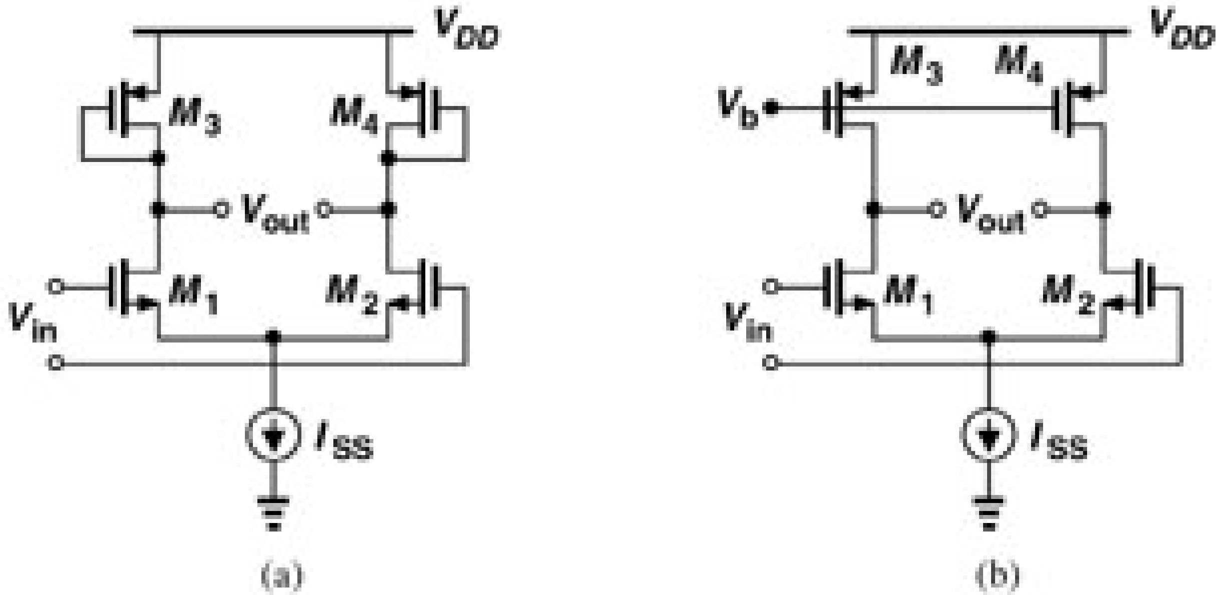
- CMRR



$$CMRR = \frac{g_{m1} + g_{m2} + 4g_{m1}g_{m2}R_{SS}}{2\Delta g_m}$$

$$\approx \frac{g_m}{\Delta g_m}[1 + 2g_m R_{SS}]$$

4.4 Differential pair with MOS Load

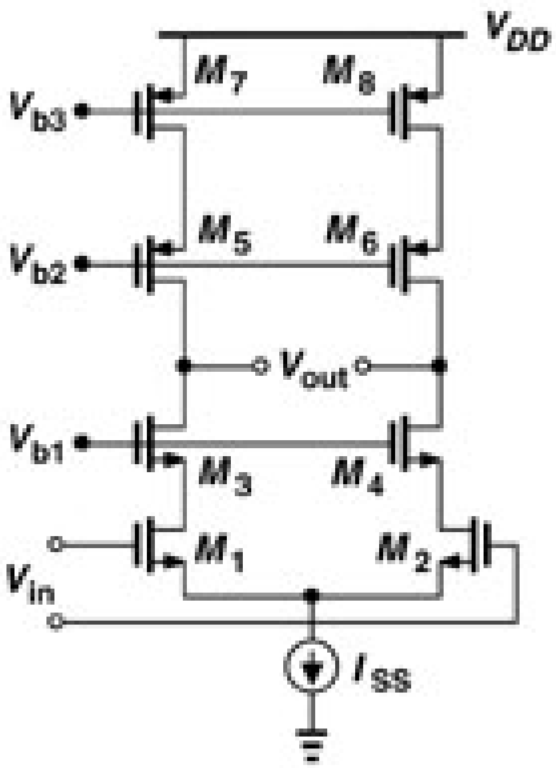


$$A_{V,diff} = -g_{mN}(g_{mP}^{-1} \| r_{oN} \| r_{oP})$$

$$\approx -\frac{g_{mN}}{g_{mP}} = -\sqrt{\frac{\mu_n (W/L)_N}{\mu_p (W/L)_P}}$$

$$A_{V,diff} = -g_{mN}(r_{oN} \| r_{oP})$$

Solution to low-gain problem: Cascoding



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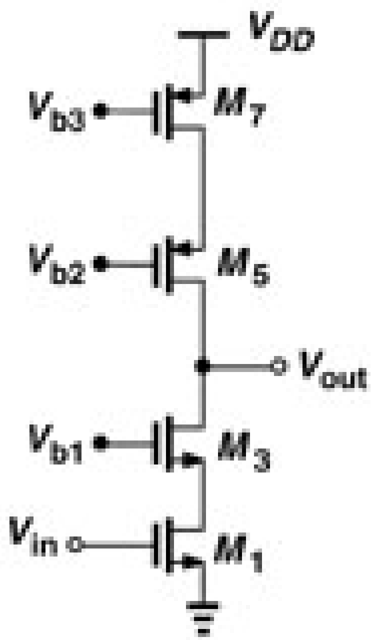

(b)

$$A_{V,diff} \approx g_{m1}[(g_{m3}r_{o3}r_{o1}) \| (g_{m5}r_{o5}r_{o7})]$$